

Topics:

1) Rounding	5) Reading Sheet Music	9) Buying a TV (ratios)
2) Estimation using significant digits	6) Music Royalties (price per play)	10) Percentages
3) Fermi Estimation (powers of ten)	7) Beyoncé (comparing statistics)	11) Sales Tax
4) Estimating Crowds	8) Feel Good Index	12) Currency conversion (proportions)

1) When rounding whole numbers there are two rules to remember:

First, you must understand the term "rounding digit." When asked to round to the closest 10, your rounding digit is the second number from the right, or 10's place, when working with whole numbers.

When asked to round to the nearest hundredth, the second place to the right of the decimal is the rounding digit.

Rules for Rounding Whole Numbers

Determine what your rounding digit is and look to the right side of it.

- If the digit is 0, 1, 2, 3, or 4, do not change the rounding digit. All digits that are on the right-hand side of the requested rounding digit will become 0.
- If the digit is 5, 6, 7, 8, or 9, your rounding digit rounds up by one number. All digits that are on the right-hand side of the requested rounding digit will become 0.

Rounding Rules for Decimal Numbers

Determine what your rounding digit is and look to the right side of it.

- If that digit is 4, 3, 2, or 1, simply drop all digits to the right of it.
- If that digit is 5, 6, 7, 8, or 9 add one to the rounding digit and drop all digits to the right of it.

2) What are significant digits? Well, they're sort of the "interesting" or "important" digits. (They're sometimes also called "significant figures".) They are the digits which give us useful information about the accuracy of a measurement. You may be asked to estimate a problem using significant figures in your calculations or only as you estimate an answer. Here are some examples of how to count the number of significant digits:

ex1-3.14159 has six significant digits. That is to say, all the numerals ("digits") give us useful information.

ex2-1000 has one significant digit: only the 1 is interesting (only it tells us anything specific); we don't know anything for sure about the hundreds, tens, or units places; the zeroes may just be placeholders; they may have rounded something off to get this value.

ex3-1000.0 has five significant digits: the ".0" tells us something interesting about the presumed accuracy of the measurement being made; namely, that the measurement is accurate to the tenths place, but that there happen to be zero tenths.

ex4-0.00035 has two significant digits: only the 3 and 5 tell us something; the other zeroes are placeholders, only providing information about relative size.

ex5-0.000350 has three significant digits: the last zero tells us that the measurement was made accurate to that last digit, which just happened to have a value of zero.

ex6-1006 has four significant digits: the 1 and 6 are interesting, and we have to count the zeroes, because they're between the two interesting numbers.

ex7-560 has two significant digits: the last zero is just a placeholder.

3) In a Fermi estimation is all that matters is getting in the right ballpark; that is, the answer should have about the right number of digits. In Fermi estimation, you can round all your numbers to the nearest order of magnitude (power of ten, 1, 10, 100, 1000...). It is usually a good strategy to decompose the problems into smaller calculations. Sometimes you can give an estimate in one step, within a factor of 10. (How much does a new compact car cost? \$10,000.) But in most cases, you'll need to break the problem into several pieces, estimate each of them, and then recombine them.

Example: How much does the New York state government spends on K-12 education every year?

The state's population is 100 million

1/10th of the population attend K-12 public schools

\$10,000 is spent per child

$$=(100,000,000)(1/10)(10000)=100,000,000,000 \text{ or } 10^{11} \text{ or } 100 \text{ billion dollars}$$










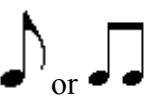

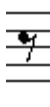

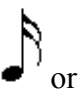



[actual=53 billion so we are in the right power of ten)

4) It has been determined that in a loose crowd, where each person is about an arm's length from their nearest neighbor, one person would occupy 10 square feet of space as an estimate. In a denser crowd, each person occupies 4.5 square feet, and in a mosh-pit density crowd, each person occupies 2.5 square feet. So, if you knew the area a crowd was covering and used these estimates for the density of the crowd, simply multiplying area by density would give you an estimate of the crowd size.

Example: Estimate the number of people in an area 500 Sq ft at a parade.

$$500\text{sqft}(1\text{ppl}/2.5\text{sqft})=200 \text{ people}$$

5) The time signature is indicated at the beginning of the measure using two numerals: The upper number indicates the number of beats in the measure, 3 in this case. The lower number indicates the note that will occupy one beat. Therefore the time signature **3/4** indicates that the measure has 3 times and that each beat is occupied by a quarter note. In **6/8** time signature the eighth note has 1 beat and the chart values are adjusted.

Note	Rest	Common Name	Musical Name	Beats in 4/4 time	IF Dotted note in 4/4
		whole note	semibreve	4	 $4 + 2 = 6$
		half note	minim	2	 $2 + 1 = 3$
		quarter note	crotchet	1	 $1 + 1/2 = 1 \frac{1}{2}$
 or 		eighth note	quaver	1/2	 $1/2 + 1/4 = 3/4$
 or 		sixteenth note	semiquaver	1/4	 $1/4 + 1/8 = 3/8$

6) Use the concept of a unit rate and whatever data is available to find an income per play rate. When rates are expressed as a quantity of 1, such as **2 feet per second** or **5 miles per hour**, they are called unit rates. If you have a multiple-unit rate such as 120 students for every 3 buses, and want to find the single-unit rate, write a ratio equal to the multiple-unit rate with 1 as the second term.

1. To establish the “per play allocation,” you then take [remaining revenue / total # service plays in that month].
2. Each publisher (the people who represent the compositions) then gets a lump sum payout of [per play allocation * total # plays publisher owns].
3. The publisher then delivers royalties to artists and songwriters; it is incumbent on the publisher to figure out how to split up their lump-sum payment to individual owners, and they also take a cut for the administration service.

7) When given two artists to compare be able to take comparable statistics and give an overall difference and also a percentage difference.

Tupac	Biggie
Studio albums <ul style="list-style-type: none"> • <i>2Pacalypse Now</i> (1991) • <i>Strictly 4 My N.I.G.G.A.Z...</i> (1993) • <i>Me Against the World</i> (1995) • <i>All Eyez on Me</i> (1996) 	Studio albums <ul style="list-style-type: none"> • <i>Ready to Die</i> (1994) • <i>Life After Death</i> (1997)

Tupac had 2 more albums than Biggie. Tupac had 150% more albums than Biggie.

8) The FGI was modeled to determine how “happy” or feel good a song would make you.

Feel Good Index: $60 + (0.00165 * (\text{BPM} - 120))^2 + (4.376 * \text{Major}) + 0.78 * \text{nChords} - (\text{Major} * \text{nChords}) + (\text{\#of positive words or notions in lyrics})$

BPM=Beats per minute, Major: 1=major, 0=minor, nChords=#of different chords

**Be able to use order of operations and the parenthesis present to calculate the FGI.

9) TVs are measured on the diagonal so you need a ratio and the Pythagorean theorem to figure out area. Widescreen ratio is 16:9 and Standard TV ratio is 4:3.

Example: 50” Widescreen TV

$$\begin{aligned}
 (16x)^2 + (9x)^2 &= 50^2 & x &= \sqrt{7.42} = 2.72 \\
 256x^2 + 81x^2 &= 2500 & 16x &= 16(2.72) = 43.52 \\
 337x^2 &= 2500 & 9x &= 9(2.72) = 24.48 \\
 x^2 &= 2500/337 = 7.42 & A &= b \cdot h = 43.52 * 24.48 = 1065.37 \text{sqin}
 \end{aligned}$$

10) A **percentage** is a proportion that shows a number as parts per hundred. The symbol '%' means 'per cent'. 9% means 9 out of every 100, or 9/10.

PERCENT -- the number with the percent sign (%).

PART -- the number with the word *is*.

WHOLE -- the number with the word *of*.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

75% of what number is 3? (or 3 is 75% of what number?)

The PERCENT *always* goes over 100.

(It's a part of the whole 100%.)

3 appears with the word *is*:

It's the PART and goes on the top.

$$\frac{3}{\text{whole}} = \frac{75}{100}$$

We're trying to find the missing WHOLE (on the bottom).

In a proportion the cross-products are equal: **So 3 times 100 is equal to 75 times the WHOLE.**

The missing WHOLE equals 3 times 100 divided by 75.

(Multiply the two opposite corners with numbers; then divide by the other number.)

$$3 \text{ times } 100 = 75 \text{ times } \textit{the whole}$$

$$300 = 75 \text{ times } \textit{the whole}$$

$$300/75 = 75/75 \text{ times } \textit{the whole}$$

$$4 = \textit{the whole}$$

$$\frac{3}{\text{---}} = \frac{75}{100}$$

(A red line connects the 3 to the 100, and the 75 to the denominator line. The 75 is circled in red.)

To **increase** or **decrease** an amount by a percentage, first calculate the percentage of the amount and then either **add** this answer on to increase the quantity, or **subtract** this answer to decrease the quantity. Also you can consider what the overall percentage would be after the figure has had its percentage increase or decrease added or subtracted. Then convert this amount to a decimal, before finally multiplying by the number in question.

11) Sales tax is a tax imposed on customers when they purchase products and services. **Multiply the cost of an item or service by the sales tax in order to find out the total cost.** The equation looks like this: *Item or service cost x sales tax (in decimal form) = total sales tax.* Add the total sales tax to the Item or service cost to get your total cost. Steps:

Change the sales tax into decimal form. For instance: 7.5% sales tax becomes .075 in decimal form

Formula: Item or service cost x sales tax (in decimal form) = total sales tax.

Sample calculation: \$60 (item cost) x .075 (sales tax) = \$4.50 total sales tax

12) An exchange rate is how much it costs to exchange one currency for another. Exchange rates fluctuate constantly throughout the week as currencies are actively traded. If the USD/CAD exchange rate is 1.0950, that means it costs 1.0950 Canadian dollars for 1 U.S. dollar. The first currency listed (USD) always stands for one unit of that currency; the exchange rate shows how much of the second currency (CAD) is needed to purchase that one unit of the first (USD). This rate tells you how much it costs to buy one U.S. dollar using Canadian dollars. To find out how much it costs to buy one Canadian dollar using U.S. dollars use the following formula: 1/exchange rate. In this case, 1 / 1.0950 = 0.9132. It costs 0.9132 U.S. dollars to buy one Canadian dollar. This price would be reflected by the CAD/USD pair; notice the position of the currencies has switched.

Example:

Suppose that the EUR/USD exchange rate is 1.20 and you'd like to convert \$100 U.S. dollars into [Euros](#). To accomplish this, simply divide the \$100 by 1.20 and the result is the number of euros that will be received: 83.33 in that case. Converting euros to U.S. dollars involves reversing that process by multiplying the number of Euros by 1.20 to get the number of U.S. dollars.