AQR UNIT 7
NETWORKS AND GRAPHS:
Circuits, Paths, and Graph Structures
Packet #___

BY: ___________________________
Introduction to Networks and Graphs:

Try drawing a path for a person to walk through each door exactly once without going back through any door more than one time.

Try 1: _____________________  Try 2: _____________________

Creating efficient routes for the delivery of goods and services (such as mail delivery, garbage collection, police patrols, newspaper deliveries, and late-night pizza deliveries) along the streets of a city, town, or neighborhood has long been a problem for city planners and businesses alike. These types of management science problems are known in mathematics as Euler circuit problems.

Euler circuit problems can all be tackled by means of a single unifying mathematical concept—the concept of a graph. The most common way to describe a graph is by means of a picture. The basic elements of such a picture are: a set of “dots” called the vertices of the graph and a collection of “lines” called the edges of the graph.

In an Euler circuit problem, by definition every single one of the streets (or bridges, or lanes, or highways) within a defined area (be it a town, an area of town, or a subdivision) must be covered by the route exactly once, ending at the location used as the start.

Here is a selection of graphs. Are they the same or different than each other? What makes a graph distinct from another?

Parts of a graph: Label each graph with its unique feature.
**Modeling graphs:** In the space below draw a graph of the following scenario: You and 3 friends all live in different houses in the same neighborhood. Each house is connected by roads to each of the other houses.

**Counting graphs:** A graph can be identified by its edges and vertices and therefore, by the degree of each vertex point. Draw a graph with only even degree vertices and a graph with 2 odd vertices and the rest even. Then count the degrees of each vertex in the picture on the right.

**Walks, Paths, Circuits:**

a) Trace the picture at the right without picking up your pencil.

b) Trace the picture at the right without picking up your pencil or retracing any steps (edges).

c) Trace the picture at the right without picking up your pencil or retracing any steps (edges), ending where you started.

**Euler Circuits:** Attempt to trace the following shapes without lifting your pencil or retracing any steps (edges) and ending where you started.
Fill out the chart based on the previous shapes. For this chart, traceable means graphs that have an Euler circuit

<table>
<thead>
<tr>
<th>Diagram #</th>
<th>Traceable?</th>
<th># of Odd Vertices</th>
<th># of Even Vertices</th>
<th>Total Vertices</th>
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Königsberg Bridge:
The following figure shows the rivers and bridges of Königsberg. Residents of the city occupied themselves by trying to find a walking path through the city that began and ended at the same place and crossed every bridge exactly once. If you were a resident of Königsberg, where would you start your walk and what path would you choose? Note, the river cuts the city in half, so one cannot travel “outside” of the picture to get from the bottom half to the top half.

Represent the Königsberg Bridge as a graph:
Problems: Complete the following problems.
1) What about when you visit the Eastern and Western wildflower gardens that have fabulous sculptures in addition to beautiful flowers along the walkways. You want to see each display without backtracking (seeing something you have already seen). Where would you start your walk and what path would you choose?

2) Your friend Chet calls you on his cell phone and tells you that he has discovered a large rock embedded with gems! He is somewhere in your favorite hiking area, which has many interconnected paths, as shown below. Chet does not know exactly where he is, but he needs your help to carry the rock. To find him, you decide it would be most efficient to jog along all the paths in such a way that no path is covered twice. Find this efficient route on the map below or explain why no such route exists.
3) You have been hired to paint the yellow median stripe on the roads of a small town. Since you are being paid by the job and not by the hour, you want to find a path through the town that traverses each road only once. In the map of the town’s roads below, find such a path or explain why no such path exists.

![Map of the town's roads](image)

4) Is it possible to draw a path for a person to walk through each door exactly once without going back through any door more than one time? If so, show the path then determine if it is possible to do so and end back at the place you started.

![Map of the rooms](image)
Follow-up Questions:
1) Define the following graph theory terms using complete sentences.

   Vertex (Vertices):

   Edges:

   Graph:

   Path:

   Circuit:

   Eulerian Path and Eulerian Circuit:

2) Form a conjecture about how you might quickly decide whether a graph has an Euler circuit, and explain why your conjecture seems reasonable. What does your conjecture tell you about the Königsberg Bridge problem and the garden scenario?

3) REFLECTION: For what situation(s) is it satisfactory to have only a path exist and not a circuit?
Part 2:

For each of these, write the valence number next to each vertex, then tell how many vertices are odd (odd valence #), and how many are even (the valence # is even).

# odd V:__ 2 ___
# odd V: ______
# odd V: ______
# even V:__ 2 __
# even V: ______
# even V: ______
# odd V: ______
# odd V: ______
# odd V: ______
# even V: ______
# even V: ______
# even V: ______

Put it together:

3 of the graphs have Euler circuits. How many odd vertices do they have?

3 of the graphs have Euler paths. How many odd vertices do they have?

3 of the graphs are not traceable. How many odd vertices do they have?

Read the rest of the explanation on the web, and then do the quiz practice.